



Paper Type: Research Paper

An Intuitionistic Fuzzy Extension to RAMS-RATMI Methods for Optimizing Electrical Discharge Machining Processes

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Citation:

Received: 27 March 2024

Revised: 10 May 2024

Accepted: 06 June 2024

Chatterjee, S., & Chakraborty, Sh. (2025). An intuitionistic fuzzy extension to RAMS-RATMI methods for optimizing electrical discharge machining processes. *Journal of fuzzy extension and applications*, 6(1), 71–93.

Abstract

The rising demand for materials with superior mechanical properties has motivated the engineering of several high-strength, heat-resistant alloys. To overcome the drawbacks of conventional machining methods, Electrical Discharge Machining (EDM) turns out to be a more feasible way of cutting such materials. However, improper setting of its different input parameters may severely affect the surface integrity of the machined parts and cause excessive tool wear. Multi-Criteria Decision-Making (MCDM) approaches have emerged as competent mathematical tools capable of handling multiple input factors and their interactions with numerous conflicting responses to figure out the ideal EDM process parameter values. In this paper, two recently introduced MCDM methods, namely Ranking Alternatives by Median Similarity (RAMS) and Ranking Alternatives by Trace to Median Index (RATMI), integrated with intuitionistic fuzzy sets (IFSs) for taking into account the uncertainty inherent in the opinions of different stakeholders, are proposed in a single framework to optimize two EDM processes. For the first EDM process, the ideal combination of different input factors is derived from discharge current = 3 A, pulse-on time = 10 μ s, pulse-off time = 5 μ s, and copper as the tool material. On the other hand, for the second process, there is a tie between two combinations of EDM parameters, i.e. peak current = 10 A, pulse-on time = 500 μ s, and gap voltage = 45 V; and peak current = 10 A, pulse-on time = 1000 μ s, and gap voltage = 50 V. Furthermore, comparative analyses against other well-known MCDM tools and sensitivity analysis studies by varying the importance of responses are also conducted for both the processes validating reliability and consistency of the ranks obtained using the proposed IF-RAMS and IF-RATMI approaches.

Keywords: Electrical discharge machining, MCDM, Optimization, RAMS, RATMI, Intuitionistic fuzzy sets.

1 | Introduction

To meet the growing industrial demands for harder and stronger materials, advances in material sciences propagated the development of several materials with advanced metallurgical and mechanical traits, making

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 <https://doi.org/10.22105/jfea.2024.450105.1427>

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them challenging to machine. Such advanced materials play a crucial role in various sectors, such as aviation and automobile industries. However, the conventional machining processes are incapable of cutting those materials, such as advanced ceramics, composites, titanium alloys, etc. Therefore, there is an increasing demand for non-traditional machining techniques that can efficiently and economically machine those advanced materials for various engineering applications [1]. Electrical Discharge Machining (EDM), being a prominent and industrially accepted non-traditional machining process, is used for cutting different difficult-to-machine materials with complex shape geometries and closer dimensional tolerances.

EDM removes material through thermal erosion by the repeated striking of electrical discharges induced between the workpiece and the tool acting as electrodes, placed within a dielectric environment when the voltage applied exceeds a certain threshold. The controlled electrical sparks cause material erosion on the workpiece surface through melting and evaporation [2]. Broadly, there are two major approaches to EDM, e.g., wire-cut and die-sinking EDM. During the die-sinking EDM operation, the geometry of the tool is replicated on the workpiece, whereas wire-cut EDM utilizes a metallic wire as an electrode and generates the desired shape or profile in the workpiece [3].

The EDM process overcomes several drawbacks of the conventional machining methods, such as chatter, vibration, and mechanical forces on the tool and workpiece, due to its non-contact mechanism of material removal [4]. As material removal in EDM occurs through thermal erosion, materials with a more comprehensive range of hardness and toughness can be machined using tools of softer materials, provided that they are electrically conductive [5]. However, the high temperature generated in the machining zone due to the electrical sparks causes several defects, such as micro-cracks, porosity, residual stresses, recast layer, heat affected zone, etc., on the workpiece face subjected to the EDM operation that may deteriorate the surface finish of the components. Post-processing operations are often carried out to achieve the desired surface quality, which may increase production costs. Compared to the conventional machining processes, EDM is slower, particularly during finishing operations [6].

Furthermore, there is a considerable amount of tool wear during the EDM process as the electrical sparks that remove material from the workpiece surface through thermal erosion simultaneously erode material from the tool electrode. Therefore, it is evident that the performance of the process with reference to material removal rate (MRR), the workpiece surface quality, Tool Wear Rate (TWR), etc., significantly depends on its various input factors, like discharge current (I), gap voltage (V_g), pulse-on time (T_{ON}), duty cycle, pulse-off time (T_{OFF}), the dielectric and its pressure, type of tool electrode used, etc. [7]. Fig. 1 outlines a schematic representation of an EDM process.

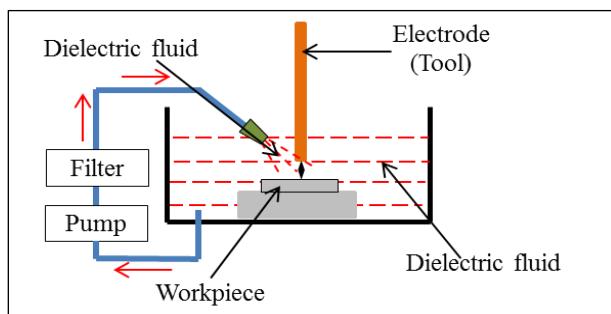


Fig. 1. Schematic representation of an EDM experimental setup.

During EDM operation, increasing peak current would increase MRR as higher pulse energy enables faster melting and vaporisation of workpiece material. However, the rise in peak current and pulse energy is accompanied by the formation of larger craters during material removal, causing higher Surface Roughness (SR) of the machined components [8]. Furthermore, a rise in voltage would enable the discharges to strike the workpiece surface more intensely, resulting in a more significant number of cracks, responsible for the poor surface integrity of the workpiece.

Similarly, recast layer thickness would also increase with increasing voltage as the dielectric would be unable to flush away larger particles of the molten material generated due to higher voltage [9]. Moreover, the thermal conductivity of the tool and workpiece electrode materials plays a significant part in the proper dissemination of the heat caused by the electrical discharges that may otherwise damage the surface integrity. Therefore, it is evident that the value of each EDM parameter would neither be too high nor too low to have a balanced compromise between productivity, product quality, and process economy.

It is thus necessary to achieve an optimal parametric setting that can simultaneously maximize material removal while ensuring better surface quality and minimum process cost. Hence, parametric optimization of an EDM process has become ardently necessary to ensure its feasibility and low-cost industrial application.

Optimization of an EDM process is mathematically complex as influences of multiple input parameters must be considered against several conflicting responses (outputs). The various methods of Multi-Criteria Decision-Making (MCDM) are viable mathematical devices for the multi-objective optimization of EDM experiments. They can identify the combination of the input parameters under consideration, which leads to the best results. Realistic manufacturing environments involve multiple stakeholders, such as process engineers, manufacturers, and customers (end-users), each having varying opinions and interests.

Depending on their priorities, the considered responses may be evaluated with dissimilar levels of importance by the decision-making experts. Traditional MCDM approaches are incapable of handling scenarios where a considerable amount of uncertainty is associated with the available information, and decision-makers are limited to vague evaluations represented by linguistic variables, such as bad, medium, good, etc. The aggregation of fuzzy sets with conventional MCDM methods, referred to as fuzzy MCDM, enables the representation of such linguistic evaluations as fuzzy numbers to derive more reliable results [10].

Zadeh [11] proposed the fuzzy set theory in 1965 as a solution for representing uncertainty in knowledge. The suitability of fuzzy sets in solving various mathematical and engineering problems has motivated researchers to advance the theory and create extensions to eliminate the drawbacks of the traditional fuzzy set theory. Notably, triangular [12], trapezoidal [13], and Intuitionistic Fuzzy Sets (IFSs) [14], as well as Neutrosophic [15], Pythagorean [16], Fermatean [17], picture [18], and hesitant fuzzy sets [19] have been integrated with traditional MCDM methods to develop their fuzzy extensions for applications in medical diagnosis, energy policy-making, industrial equipment and fund selection, among other domains.

Therefore, integrating MCDM methods with fuzzy sets enables the consideration of subjectivity in the judgment of the stakeholders in decision-making scenarios. Furthermore, the use of IFSs overcomes the shortcomings of conventional fuzzy sets in handling the vagueness associated with data in real-world applications. Therefore, this paper puts forth the implementation of the Ranking Alternatives by Median Similarity (RAMS) and the Ranking Alternatives based on Trace Median Index (RATMI) MCDM methods, integrated with an Intuitionistic Fuzzy (IF) environment to obtain the optimal combination of process parameters for two EDM operations based on historical data.

Fig. 2 depicts the methodology followed for optimizing an EDM process using IF-RAMS and IF-RATMI techniques in the form of a flowchart. The rest of the paper is ordered as follows: A concise survey of the prior literature on the use of various mathematical tools, mainly MCDM methods, for searching for the ideal combination of parameters for EDM processes is described in Section 2. Section 3 summarizes a background on the mathematical foundation of the RAMS and RATMI techniques in an IF environment. Section 4 demonstrates applications of IF-RAMS and IF-RATMI approaches for optimizing the considered EDM processes, and Section 5 concludes the paper.

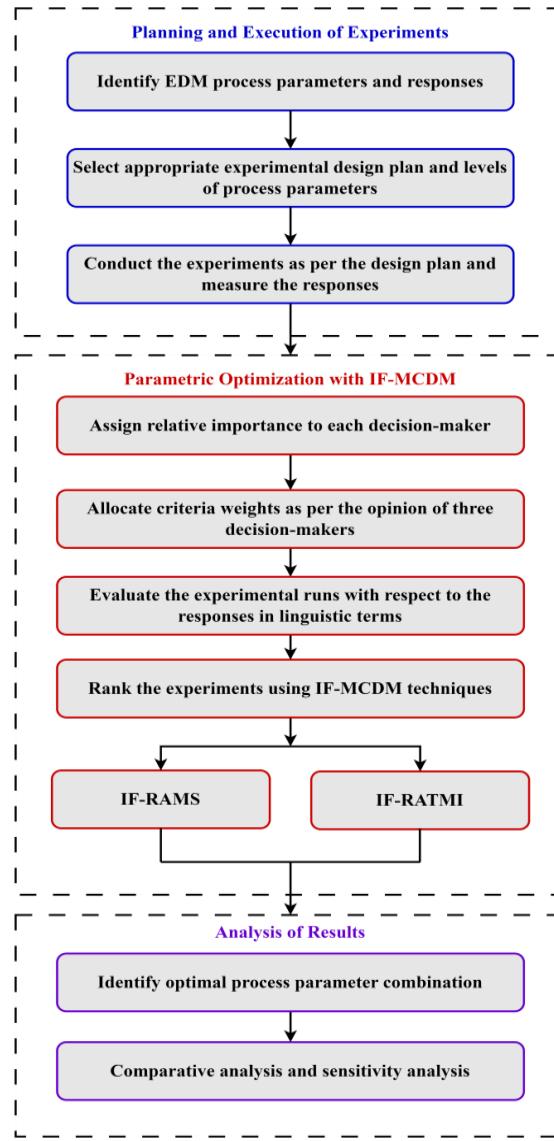


Fig. 2. Flowchart outlining the proposed approach for parametric optimization of an EDM process.

2 | Literature Review

In the following section, a brief survey of the existing literature concerned with the applications of diverse mathematical/MCDM tools for optimizing EDM processes is presented. Sidhu and Yazdani [20] compared the relative performance of the Desirability Function Approach (DFA) and Lexicographic Goal Programming (LGP) while optimizing MRR, TWR, and mean residual stress during EDM operation of SiC/A359 composite materials. Pradhan [21] employed a Central Composite Design (CCD) to conduct the EDM experimental trials and, subsequently, applied the Response Surface Methodology (RSM) as well as Grey Relational Analysis (GRA) methods to optimize the process parameters.

Furthermore, the authors in [21] integrated the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) with Shannon's entropy technique of prioritizing the evaluation criteria to solve the problem above. Using Taguchi's L_{16} Orthogonal Array (OA), Kumar and Rai [22] studied the dependence among factors, such as I , T_{ON} and T_{OFF} and performance measures, like SR, MRR, and TWR during the EDM of AA7050-10 (wt.)% B₄C composites. Furthermore, the authors also adopted two MCDM tools, i.e., GRA and Additive Ratio ASsessment (ARAS), to seek out the desirable values of the EDM parameters. Singh et al. [23] utilized

TOPSIS to maximize the MRR while minimizing the TWR and Electrode Wear Rate (EWR) during EDM of Inconel 718 material while treating I , T_{ON} and dielectric flushing pressure as the input parameters.

A similar study was also performed by Sharma et al. [24] using a Central Composite Face-Centred Design (CCFCD) plan to compute the ideal setting of EDM process parameters utilizing the GRA and RSM techniques, integrated with grey Teaching-Learning-Based Optimization (TLBO) method and developed a hybrid MCDM approach. Similarly, Huu [25] applied a hybrid Taguchi-based Analytic Hierarchy Process (AHP)-Deng's similarity approach to achieve optimal performance during the powder-mixed EDM operation of die steels.

Considering I , T_{ON} , and V_g as the process parameters, Karthik Pandeyan et al. [26] optimized MRR, Circularity error (CIR), Cylindricity error (CYL), and TWR of AA6061-T6/15 (wt.%) SiC aluminium metal matrix composites (Al-MMCs) utilizing the Combinative Distance-based ASsessment (CODAS) method. Bhattacharjee et al. [27] conducted EDM of Al-Cu-SiCp composite materials treating I , T_{ON} , and percentage of reinforcement as the input factors to measure the MRR, SR, and EWR as performance metrics. The authors later tested the Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) to identify optimal combinations of process parameters. A similar study on EDM of composite materials was also carried out by Ganesan et al. [28], where COmplex PROportional ASsessment (COPRAS) was employed to compute the ideal levels of I , T_{ON} , and V_g to simultaneously optimize the MRR, EWR, CIR and CYL during the EDM of AA6082/3(wt.%)BN/1(wt.%)MoS₂. Goswami et al. [29] applied COPRAS integrated with ARAS to evaluate the ideal values of input parameters during the EDM of high-carbon chromium tool steels.

More recent studies, such as Phan et al. [30], investigated the impacts of V_g , I , and T_{ON} on MRR and SR during EDM of SKD61 die steel. The authors applied the Data Envelopment Analysis-based Ranking (DEAR) method and calculated the optimal values of the EDM parameters. Gopalakrishnan et al. [31] simultaneously optimized MRR, SR, TWR, Micro-Hardness (MH), and Dimensional Depth (DD) during EDM of Ph15-5 stainless steel workpieces. The authors in [31] proposed a genetic algorithm (GA)-TOPSIS integrated MCDM algorithm to search for the most desirable levels of EDM parameters.

Kumar et al. [32] investigated the impact of the tool electrode material on response features, such as MRR, TWR, SR, and CIR, while machining pearlitic SG iron through a Step-wise Weight Assessment Ratio Analysis (SWARA)-integrated Combined Compromise Solution (CoCoSo) algorithm. Biswal et al. [33], Rajmohan et al. [34], and Tolcha and Lemu [35] endeavoured to search the optimal combinations of input parameters for EDM of Al-MMCs using several widely-used MCDM tools, like DEAR, GRA, VIKOR (Vlse Kriterijumska Optimizacija I Kompromisno Resenje in Serbian), and Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS).

Table 1 summarizes the existing literature related to the parametric optimization of EDM processes. This table describes information concerning the workpiece material machined, the design of experiments used, the EDM input factors and responses considered, and different MCDM tools utilized for optimizing the EDM processes. The summary highlights that the past researchers have mainly conducted EDM experiments using Taguchi's OA design plan on different hard-to-cut engineering materials, primarily considering T_{ON} , T_{OFF} , I , and V_g as the process parameters, and commonly studying response metrics, such as MRR, SR, and TWR/EWR.

Due to the versatility of MCDM tools and their fuzzy extensions, several recent studies have demonstrated their applicability in various research sectors. Ali et al. [36] presented a hybridization of the Intuitionistic Fuzzy Soft Set (IFSS) with Aczel-Alsina t-norm and t-conorm operators to develop aggregation operations, such as sum and product laws, to handle problems where the evaluations are provided in terms of Intuitionistic Fuzzy Soft Values (IFSVs). Isabels et al. [37] proposed a trapezoidal fuzzy extension to the VIKOR method to evaluate and rank the digital marketing capability of four metaverse platforms.

Based on a comprehensive survey of the current literature, it is identified that most of the previous literature focuses on applying the Taguchi-GRA and TOPSIS methods for optimizing EDM processes. Furthermore, there is very limited work exploring the applicability and potentiality of newer MCDM tools and fuzzy-MCDM methods in this domain. It is necessary to declare that no prior work focusing on the application of MCDM tools integrated into an IF environment to compute the ideal level of process parameters in EDM operations exists in the available literature, to the best of the authors' knowledge. Hence, to address these existing knowledge gaps, this paper contributes the following:

- I. Two recently published MCDM methods, RAMS and RATMI, developed based on Ranking Alternatives by Perimeter Similarity (RAPS) and Multiple Criteria Ranking by Alternative Trace (MCRAT), respectively, are adopted to demonstrate their potential in optimizing EDM processes through two illustrative examples. To the best of our knowledge, applications of RAMS and RATMI in the existing literature have only been restricted to solving a material selection problem in the automobile industry. There has been no previous effort to apply these two MCDM methods for the process parameter optimization of any machining operation.
- II. This paper proposes IF extensions to both these methods for the first time to efficiently handle the uncertainty in decision-making that arises in the manufacturing environment due to the subjective opinions of the stakeholders involved. It is also worth mentioning here that no previous application of IF-MCDM methods for optimizing EDM parameters exists in the present literature.

Table 1. Summary of literature review on optimization of EDM processes.

Author(s)	Work Material(s)	Design	EDM Parameters	Responses	Tool(s)
[20]	SiC/A359	L_9 OA	I, T_{ON}, T_{OFF}	MRR, TWR, mean residual stress	DFA, LGP
[21]	AISI D2 tool steel	CCD	I, T_{ON} , duty cycle, V_g	MRR, TWR, radial overcut	GRA, TOPSIS
[22]	AA7050-10(wt.) % B ₄ C	L_{16} OA	I, T_{ON}, T_{OFF}	MRR, SR, TWR	ARAS, GRA
[23]	Inconel 718	L_9 OA	I, T_{ON} , flushing pressure	MRR, TWR, EWR	TOPSIS
[24]	Titanium	CCFCD	I, T_{ON}, T_{OFF}	Drilling rate, EWR	GRA
[25]	SKD61, SKD11, SKT4	L_{27} OA	Tool material, polarity, I, T_{ON}, T_{OFF} , powder concentration	MRR, TWR, SR, MH, white layer thickness	Hybrid Taguchi-AHP-Deng's method
[26]	AA6061-T6/15 (wt.) % SiC	L_{27} OA	I, T_{ON}, V_g	MRR, TWR, CIR, CYL	CODAS
[27]	Al-Cu-SiCp composite	L_9 OA	I, T_{ON} , % of reinforcement	MRR, SR, EWR	PROMETHEE
[28]	AA6082/3 (wt.)% BN/1 (wt.)% MoS ₂	L_{27} OA	I, T_{ON}, V_g	MRR, EWR, CIR, CYL	COPRAS
[29]	High carbon chromium tool steel	L_9 OA	Dielectric level, I , flushing pressure, pulse duration	Aerosol concentration, relative tool wear ratio, dielectric consumption, process time, process energy	Hybrid COPRAS-ARAS
[30]	SKD61 die steel	-	I, T_{ON}, V_g	MRR, SR	DEAR
[31]	Ph14-4 SS	L_{27} OA	V_g, I, T_{ON}, T_{OFF} , tool shape, flushing pressure	MRR, SR, TWR, MH, DD	Multi-objective GA, TOPSIS

Table 1. Continued.

Author(s)	Work Material(s)	Design	EDM Parameters	Responses	Tool(s)
[32]	Pearlitic SG iron (grade 450/12)	-	Tool material	MRR, SR, TWR, CIR	SWARA-CoCoSo
[33]	Al-WC-B ₄ C hybrid composite	L ₉ OA	I, V _g , T _{ON}	MRR, SR, machining time	MARCOS
[34]	Al8081/SiC MMC	CCFCD	I, T _{ON} , T _{OFF}	MRR, TWR	DEAR
[35]	LM25Al/VC composite	CCD	I, T _{ON} , V _g	MRR, TWR, SR	TLBO, GRA, VIKOR
This paper	AZ31 Mg alloy, Ti-6Al-4V	L ₁₆ OA	I, T _{ON} , T _{OFF} , V _g , tool material	SR, MRR, TWR, OC, TC, CIR, CYL	IF-RAMS, IF-RATMI

3 | Methods

3.1 | Intuitionistic Fuzzy Sets

IFSs, propounded by Atanassov [38] in 1986, are an improvement to the traditional fuzzy sets presented by Zadeh [11]. IFSs enhance the ability of fuzzy sets to take into account the degree of uncertainty associated with the data by proposing a degree of hesitancy for each element. According to the IFS theory, we may define an IFS \tilde{A} within a finite domain X , consisting of a degree of hesitation coupled with the membership degree and non-membership degree as per the traditional fuzzy set theory, as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)) | x \in X\}, \quad (1)$$

where the membership degree, $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$, represents a measure of belongingness of an element x to IFS \tilde{A} , and the non-membership degree is given by $v_{\tilde{A}}(x): X \rightarrow [0,1]$, following the relationship $0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1$. The hesitancy degree, $\pi_{\tilde{A}}(x)$, accounts for the indecision or scepticism associated with an element defined as shown below:

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x). \quad (2)$$

According to fuzzy sets, $v_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x)$ and hence, they fail to account for the hesitation associated with any decision-making process. Therefore, IFSs are better equipped to model practical decision-making scenarios where the decision-making experts have a degree of hesitancy associated with their evaluation. Assume $\tilde{A} = \langle \mu_{\tilde{A}}, v_{\tilde{A}} \rangle$ and $\tilde{B} = \langle \mu_{\tilde{B}}, v_{\tilde{B}} \rangle$ are intuitionistic fuzzy numbers (IFNs). The standard mathematical operations performed on \tilde{A} and \tilde{B} may be defined, as shown below [39]:

$$\tilde{A} \oplus \tilde{B} = \{\mu_{\tilde{A}} + \mu_{\tilde{B}} - \mu_{\tilde{A}}\mu_{\tilde{B}}, v_{\tilde{A}}v_{\tilde{B}}\}. \quad (3)$$

$$\tilde{A} \otimes \tilde{B} = \{\mu_{\tilde{A}}\mu_{\tilde{B}}, v_{\tilde{A}} + v_{\tilde{B}} - v_{\tilde{A}}v_{\tilde{B}}\}. \quad (4)$$

$$\lambda \tilde{A} = \{1 - (1 - \mu_{\tilde{A}})^\lambda, v_{\tilde{A}}^\lambda\}. \quad (5)$$

$$\tilde{A}^\lambda = \{\mu_{\tilde{A}}^\lambda, 1 - (1 - v_{\tilde{A}})^\lambda\}. \quad (6)$$

In MCDM applications, it is often necessary to rank IFNs in an order of preference. For this purpose, the corresponding score function is defined such that the IFN with a higher score function is considered greater. The score function of an IFN can be computed as [40]:

$$S(\tilde{A}) = \frac{\mu_{\tilde{A}} - v_{\tilde{A}} + 1}{2}; S(\tilde{A}) \in [0,1]. \quad (7)$$

3.2 | IF-RAMS

The RAMS method, proposed by Abdulaal and Bafail [41], is a modification of the newly developed RAPS method. The RAPS method decomposes each alternative into two components by separating the beneficial and non-beneficial evaluation criteria. Subsequently, the beneficial and non-beneficial parts, referred to as the max and min components, are treated to be at ninety-degree angles, forming a right-angled triangle for each alternative. The RAPS approach evaluates the similarity between the perimeter of the triangles created by the decomposed components of all alternatives and the optimal one and ranks them based on a perimeter similarity index, defined as the ratio of the individual and optimal perimeters. As an extension to this theory, the RAMS method assigns a rank to the alternatives based on the similarity between the median of the triangle between the decomposed max-min components of every alternative and that of the ideal one.

The RAMS approach in an IF environment (IF-RAMS) evaluates the efficacy of the available alternatives against the responses and represents the evaluations as IFNs. A typical MCDM problem consists of a finite number of alternatives, say $A = [A_1, A_2, \dots, A_m]$ and a finite set of criteria $C = [C_1, C_2, \dots, C_n]$. The mathematical steps involved in ranking the alternatives utilizing the given criteria according to the IF-RAMS approach are outlined below:

Step 1. Evaluate available alternatives in relation to each criterion using linguistic variables.

Step 2. Transform the above-mentioned linguistic variables into their corresponding IFNs and obtain the initial IF decision matrix.

$$\tilde{X} = [\tilde{x}_{ij}]_{m \times n}; i = 1, 2, \dots, m; j = 1, 2, \dots, n, \quad (8)$$

where the IFN $\tilde{x}_{ij} = \{\mu_{ij}, v_{ij}\}$ represents the evaluation of alternative i against criterion j .

Step 3. Formulate the weighted IF decision matrix as the product of the IFN-based performance evaluation of every alternative with the corresponding IF criteria weights.

$$\tilde{U} = [\tilde{u}_{ij}]_{m \times n}; \tilde{u}_{ij} = \tilde{w}_j \otimes \tilde{x}_{ij}. \quad (9)$$

Step 4. Find the optimal alternative for each criterion (response).

$$\tilde{Q} = [\tilde{q}_j]_{1 \times n}; \tilde{q}_j = \{\mu_{\tilde{q}_j}, v_{\tilde{q}_j}\}. \quad (10)$$

$$\mu_{\tilde{q}_j} = \max_i \mu_{ij}; v_{\tilde{q}_j} = \min_i v_{ij}. \quad (11)$$

Step 5. Separate the optimal alternative into the corresponding max (beneficial) and min (non-beneficial) components of the criteria set.

$$\tilde{Q} = \tilde{Q}^{\max} \cup \tilde{Q}^{\min}. \quad (12)$$

$$\mu_{\tilde{q}_j} = \max_i \mu_{ij}; v_{\tilde{q}_j} = \min_i v_{ij}, \quad (13)$$

where k denotes the count of beneficial criteria, and h symbolizes the number of non-beneficial criteria.

Step 6. Decompose the available alternatives into their constituent max and min components.

$$\tilde{U}_i = \tilde{U}_i^{\max} \cup \tilde{U}_i^{\min}. \quad (14)$$

$$\tilde{U}_i = \{\tilde{u}_{i1}, \tilde{u}_{i2}, \dots, \tilde{u}_{ik}\} \cup \{\tilde{u}_{i1}, \tilde{u}_{i2}, \dots, \tilde{u}_{ih}\}. \quad (15)$$

Step 7. Evaluate the magnitude of max and min components for all alternatives, including the optimal alternative, using the following equations:

$$\tilde{U}_{ik} = \{\tilde{u}_{i1}^2 \oplus \tilde{u}_{i2}^2 \oplus \dots \oplus \tilde{u}_{ik}^2\}^{\frac{1}{2}}. \quad (16)$$

$$\tilde{U}_{ih} = \{\tilde{u}_{i1}^2 \oplus \tilde{u}_{i2}^2 \oplus \dots \oplus \tilde{u}_{ih}^2\}^{\frac{1}{2}}. \quad (17)$$

$$\tilde{Q}_k = \{\tilde{q}_1^2 \oplus \tilde{q}_2^2 \oplus \dots \oplus \tilde{q}_k^2\}^{\frac{1}{2}}. \quad (18)$$

$$\tilde{Q}_h = \{\tilde{q}_1^2 \oplus \tilde{q}_2^2 \oplus \dots \oplus \tilde{q}_h^2\}^{\frac{1}{2}}. \quad (19)$$

Step 8. Obtain the median for the optimal alternative.

$$\tilde{M} = \frac{(\tilde{Q}_k \oplus \tilde{Q}_h)^{\frac{1}{2}}}{2}. \quad (20)$$

Step 9. Compute the median for each alternative.

$$\tilde{M}_i = \frac{(U_{ik}^2 \oplus U_{ih}^2)^{\frac{1}{2}}}{2}. \quad (21)$$

Step 10. Determine the median similarity for every alternative as shown below:

$$MS_i = \frac{S(\tilde{M}_i)}{S(\tilde{M})}. \quad (22)$$

Higher value of MS_i is an indication of the median for the corresponding alternative being closer to the median for the optimal alternative. Therefore, the alternatives can be ranked in descending order of MS_i to ensure alternatives with the higher MS_i values receive a better rank.

3.3 | IF-RATMI

The RATMI is a combination of RAMS and MCRAT methods proposed by Urošević et al. [42]. It integrates the median similarity index with the trace of the matrix of the max and min components to obtain an aggregate measure, referred to as the majority index. The IF-RATMI approach utilizes IFNs to represent the performance of the alternatives against the considered evaluation criteria. To derive a ranking of the alternatives using the IF-RATMI approach, the IF-RAMS method must be executed first, and then the following mathematical calculations are employed:

Step 1. Formulate a diagonal matrix (F) consisting of the magnitude of the max and min components of the optimal alternative. Similarly, develop diagonal matrixes (G_i) consisting of the magnitude of the components of each alternative.

$$F = \begin{bmatrix} \tilde{Q}_k & 0 \\ 0 & \tilde{Q}_h \end{bmatrix}. \quad (23)$$

$$G_i = \begin{bmatrix} \tilde{U}_{ik} & 0 \\ 0 & \tilde{U}_{ih} \end{bmatrix}; \text{ for all } i = [1, 2, \dots, m]. \quad (24)$$

Step 2. Compute the product of matrixes F and G_i for each alternative and subsequently, estimate the trace (tr_i) of the product matrix (T_i).

$$T_i = F \times G_i = \begin{bmatrix} \tilde{t}_{11i} & 0 \\ 0 & \tilde{t}_{22i} \end{bmatrix}. \quad (25)$$

$$tr_i = S(\tilde{t}_{11i} \oplus \tilde{t}_{22i}). \quad (26)$$

Step 3. Calculate the majority index (E_i) for each alternative as the weighted sum of RAMS and MCRAT strategies.

$$E_i = \omega \frac{tr_i - tr^-}{tr^* - tr^-} + (1 - \omega) \frac{MS_i - MS^-}{MS^* - MS^-}. \quad (27)$$

$$MS^* = \max_i (MS_i); MS^- = \min_i (MS_i), \quad (28)$$

where ω is the importance assigned to the MCRAT approach, while $(1 - \omega)$ is the weight allotted to the RAMS method.

Step 4. Arrange the available alternatives in a decreasing order of the majority index (E_i).

4 | Illustrative Examples

Example 1. The experimental data from an EDM process on AZ31 magnesium alloy [43] is taken into account to demonstrate the application of both IF-RAMS and IF-RATMI approaches for searching the ideal intermix of process parameters. Utilizing Taguchi's L₁₆ OA, Somasundaram and Kumar [43] designed 16 experiments and studied the dependence of four input factors, particularly T_{ON}, T_{OFF}, I, and tool electrode material, on seven characteristics of the machined parts, e.g., SR, MRR, TWR, taper cut (TC), CIR, overcut (OC), and CYL. The EDM of AZ31 magnesium alloy workpieces (having excellent biodegradable and biocompatible properties and dimensions of 50×30×6 mm) was carried out using electrodes (10 mm in diameter and 25 mm long).

To develop the corresponding design plan, Somasundaram and Kumar [43] varied every EDM parameter across four different levels, i.e., I {3, 6, 9, 12 A}, T_{ON} {10, 20, 30, 40 μ s}, T_{OFF} {5, 6, 7, 8 μ s}, and tool material {Copper, Brass, Graphite, Graphite-Copper}. Each experimental run involved machining a 10-mm diameter hole through a die-sinking EDM machine (Ratnaparkhi ALTRA ZNC ORB 5530). The SR values of the machined surface for each experiment were recorded by a Mitutoyo SJ-201 portable SR testing apparatus, and the Mitutoyo CRYSTA-Apex S544 CNC coordinate measuring machine was utilized to measure OC, TC, CYL, and CIR of the generated holes.

For measuring the values of TWR and MRR, the measurement of weights of the tool and the workpiece, respectively, were recorded at the beginning and the end of the machining experiments using a Citizen CX 220 digital weighing balance. To avoid any inaccuracy during measurement, the average of two trials was considered for each of the responses. Somasundaram and Kumar [43] applied GRA and TOPSIS to optimize the seven above-mentioned responses simultaneously and identified the corresponding optimal parametric settings. For both GRA and TOPSIS, experimental run 1 having the combination of input parameters as I = 3A, T_{ON} = 10 μ s, T_{OFF} = 5 μ s, and copper tool material appeared as the best mixture resulting in the corresponding responses as SR = 3.20 μ m, MRR = 7.5512 mm³/min, TWR = 0.0042 g/min, TC = 0.061 mm, OC = 0.0140 mm, CYL = 0.0421 mm, and CIR = 0.0184 mm. *Table 2* provides the experimental design plan of each experimental run as well as the recorded values of the quality characteristics.

Table 2. Experimental data for Example 1 [43].

Exp. No.	I (A)	T _{ON} (μ s)	T _{OFF} (μ s)	Tool Material	SR (μ m)	MRR (mm ³ /min)	TWR (g/min)	OC (mm)	TC (mm)	CIR (mm)	CYL (mm)
E ₁	3	10	5	Cu	3.20	7.5512	0.0042	0.0140	0.061	0.0184	0.0421
E ₂	3	20	6	Br	6.68	61.1810	0.0206	0.0865	0.045	0.0115	0.0677
E ₃	3	30	7	Gr	9.89	89.4419	0.0184	0.1075	0.037	0.0080	0.0905
E ₄	3	40	8	Gr-Cu	11.31	47.5763	0.0014	0.0835	0.037	0.0068	0.0746
E ₅	6	10	6	Gr	5.00	34.8201	0.0062	0.0715	0.010	0.0410	0.1209
E ₆	6	20	5	Gr-Cu	4.96	30.4132	0.0015	0.0330	0.059	0.0074	0.0671
E ₇	6	30	8	Cu	10.02	81.8268	0.0023	0.0625	0.033	0.0735	0.0749
E ₈	6	40	7	Br	9.79	129.2052	0.0369	0.1000	0.039	0.0097	0.0763
E ₉	9	10	7	Gr-Cu	6.55	14.9400	0.0010	0.0315	0.002	0.0353	0.0640
E ₁₀	9	20	8	Gr	11.27	73.1567	0.0063	0.1603	0.023	0.0062	0.0814
E ₁₁	9	30	5	Br	11.38	131.6526	0.0429	0.1550	0.028	0.0456	0.0454
E ₁₂	9	40	6	Cu	12.76	156.8343	0.0084	0.2140	0.015	0.0023	0.0921
E ₁₃	12	10	8	Br	4.80	46.3972	0.0255	0.010	0.039	0.0043	0.0452
E ₁₄	12	20	7	Cu	8.04	33.2228	0.0010	0.1375	0.006	0.0269	0.0628
E ₁₅	12	30	6	Gr-Cu	9.00	167.2435	0.0091	0.1690	0.004	0.0311	0.0780
E ₁₆	12	40	5	Gr	14.28	202.6980	0.0116	0.2855	0.004	0.0640	0.1275

The decision-making experts (stakeholders) involved in a real manufacturing environment may have different requirements and interests. As a result, they may evaluate the responses under consideration with varying levels of importance. In such situations, assessing the importance of the responses as linguistic ratings, such as 'very important', 'medium, or 'unimportant', proves to be more suitable for the decision-making experts rather than providing crisp values for their relative importance.

Therefore, the relative priority of the criteria (responses) was evaluated in linguistic terms by three stakeholders (manufacturer, process engineer, and end-user) and then converted into IFNs as described by the scale in *Table 3*. The manufacturer prioritizes productivity, cost efficiency, and customer satisfaction and assigns the highest importance to MRR, followed by TWR and SR. The process engineer wants to ensure that there is minimal damage to the tool, that the process is economically feasible, and that the desired product geometry can be attained.

Therefore, the highest importance is allotted to TWR, followed by MRR, OC, and TC. Finally, the end-user desires the highest quality of surface finish and dimensional accuracy and, hence, assigns the maximum relative importance to SR, followed by CIR and CYL. The complete linguistic evaluation of the responses by each of the stakeholders is provided in *Table 4*. As per the scale given in *Table 3* and linguistic evaluations in *Table 4*, the relative priority of the responses is converted to IFN ratings and combined to assign a single IF priority rating (weight) to the individual responses, as shown in *Table 5*. It is important to point out that equal importance is allocated to each of the stakeholders.

Table 3. IFNs for linguistic rating of the responses for Example 1 [44].

Linguistic Variable	IFN		
	μ	ν	π
Very important (VI)	0.88	0.08	0.04
Important (I)	0.75	0.20	0.05
Medium (M)	0.50	0.45	0.05
Unimportant (UI)	0.35	0.60	0.05
Very unimportant (VUI)	0.08	0.88	0.04

Table 4. Linguistic evaluation of the responses by different stakeholders for Example 1.

Response	Manufacturer	Process Engineer	End-user
SR	I	UI	VI
MRR	VI	I	M
TWR	I	VI	UI
OC	M	I	M
TC	M	I	M
CIR	UI	M	I
CYL	UI	M	I

Table 5. Aggregated IF response weights for Example 1.

Response	μ	ν	π
SR	0.731	0.213	0.057
MRR	0.753	0.193	0.054
TWR	0.731	0.213	0.057
OC	0.603	0.343	0.053
TC	0.603	0.343	0.053
CIR	0.567	0.378	0.055
CYL	0.567	0.378	0.055

Solving for the ideal settings of the EDM process parameters using the IF-RAMS and IF-RATMI methods necessitates the evaluation of the performance of each of the experimental runs in relation to the measured responses in linguistic expressions, like extremely good, very good, bad, very bad, and so on. As per the recorded response values in *Table 2*, the performance evaluations of the alternative runs are allotted to ten different linguistic variables as per the scale described in *Table 6*. It must be noted here that the sole beneficial (higher-the-better) criterion is MRR, while the rest are non-beneficial (lower-the-better) criteria. Therefore, the linguistic evaluation is performed to ensure that a better linguistic rating corresponds to increasing values

of MRR and decreasing values of SR, TWR, OC, TC, CIR, and CYL. *Table 7* provides the corresponding IFNs for evaluating the experiments against the responses as linguistic variables.

Table 6. Scale for linguistic evaluation of the responses for different experiments in Example 1.

Linguistic Variable	SR	MRR	TWR	OC	TC	CIR	CYL
Extremely good (EG)	SR≤4.308	MRR>183.183	TWR≤0.00519	OC≤0.03755	TC≤0.00779	CIR≤0.00942	CYL≤0.05064
Very very good (VVG)	4.308<SR≤ 5.416	163.668<MRR≤183.183	0.00519<TWR≤0.00938	0.03755<OC≤0.0651	0.00779<TC≤0.0138	0.00942<CIR≤0.01654	0.05064<CYL≤0.05918
Very good (VG)	5.416<SR≤ 6.524	144.154<MRR≤163.668	0.00938<TWR≤0.01357	0.0651<OC≤0.09265	0.0138<TC≤0.0197	0.01654<CIR≤0.02366	0.05918<CYL≤0.06772
Good (G)	6.524<SR≤ 7.632	124.639<MRR≤144.154	0.01357<TWR≤0.01776	0.09265<OC≤0.1202	0.0197<TC≤0.0256	0.02366<CIR≤0.03078	0.06772<CYL≤0.07626
Medium good (MG)	7.632<SR≤ 8.74	105.125<MRR≤124.639	0.01776<TWR≤0.02195	0.1202<OC≤0.14775	0.0256<TC≤0.0315	0.03078<CIR≤0.0379	0.07626<CYL≤0.0848
Medium (M)	8.74<SR≤9.848	85.609<MRR≤105.125	0.02195<TWR≤0.02614	0.14775<OC≤0.1753	0.0315<TC≤0.0374	0.0379<CIR≤0.04502	0.0848<CYL≤0.09334
Medium bad (MB)	9.848<SR≤10.956	66.095<MRR≤85.609	0.02614<TWR≤0.03053	0.1753<OC≤0.20285	0.0374<TC≤0.0433	0.04502<CIR≤0.05214	0.09334<CYL≤0.10188
Bad (B)	10.956<SR≤12.064	46.581<MRR≤66.095	0.03033<TWR≤0.03452	0.20285<OC≤0.2304	0.0433<TC≤0.0492	0.05214<CIR≤0.05926	0.10188<CYL≤0.11042
Very bad (VB)	12.064<SR≤13.172	27.066<MRR≤46.581	0.03452<TWR≤0.03871	0.2304<OC≤0.25795	0.0492<TC≤0.0551	0.05926<CIR≤0.0638	0.11042<CYL≤0.11896
Very very Bad (VVVB)	SR>13.172	MRR≤27.066	TWR>0.03871	OC>0.25795	TC>0.0551	CIR>0.0638	CYL>0.11896

Table 7. IFN scale for evaluation of experiments as linguistic variables.

Linguistic Variable	μ	ν	π
EG	1.00	0.00	0.00
VVG	0.85	0.10	0.05
VG	0.80	0.15	0.05
G	0.70	0.20	0.10
MG	0.60	0.30	0.10
M	0.50	0.40	0.10
MB	0.40	0.50	0.10
B	0.25	0.60	0.15
VB	0.10	0.75	0.15
VVB	0.10	0.90	0.00

First, it is considered that the parametric optimization of the EDM operation on AZ31 magnesium alloy is solved using the IF-RAMS approach. Employing the scales shown in *Tables 6* and *7*, the initial IF decision matrix is formulated as provided in *Table 8*. Thereafter, by employing *Eq. (9)*, the weighted IF decision matrix is established with the help of values in *Table 8* and aggregated IF criteria weights in *Table 5*. As an example, considering the SR obtained in experimental run 1 is represented by the IFN $\langle 1,0 \rangle$ from *Table 8*, and the aggregated IF weight for SR is $\langle 0.731, 0.213 \rangle$.

Hence, the weighted IF rating using *Eq. (9)* is $\langle 1.0 \times 0.731, 0 + 0.213 - (0 \times 0.213) \rangle = \langle 0.731, 0.213 \rangle$. Considering the weighted IF performance evaluations of the alternatives against each response, the optimal alternative is computed using *Eqs. (10)* and *(11)*. The weighted IF decision matrix and the optimal experiment corresponding to each response are provided in *Table 9*. In *Table 9*, the optimal alternative under SR response is determined as $\langle \max(0.731, 0.512, 0.292, \dots), \min(0.213, 0.370, 0.606, \dots) \rangle = \langle 0.731, 0.213 \rangle$. The weighted IF evaluations of the experiments and the optimal experimental run are now decomposed into their max (beneficial) and min (non-beneficial) components, employing *Eqs. (12)-(15)*.

Table 8. Initial IF decision matrix for Example 1.

Exp. No.	SR		MRR		TWR		OC		TC		CIR		CYL	
	μ	ν												
E ₁	1.00	0.00	0.10	0.90	1.00	0.00	1.00	0.00	0.10	0.90	0.80	0.15	1.00	0.00
E ₂	0.70	0.20	0.25	0.60	0.60	0.30	0.80	0.15	0.25	0.60	0.85	0.10	0.80	0.15
E ₃	0.40	0.50	0.50	0.40	0.60	0.30	0.70	0.20	0.50	0.40	1.00	0.00	0.50	0.40
E ₄	0.25	0.60	0.25	0.60	1.00	0.00	0.80	0.15	0.50	0.40	1.00	0.00	0.70	0.20
E ₅	0.85	0.10	0.10	0.75	0.85	0.10	0.80	0.15	0.85	0.10	0.50	0.40	0.10	0.90
E ₆	0.85	0.10	0.10	0.75	1.00	0.00	1.00	0.00	0.10	0.90	1.00	0.00	0.80	0.15
E ₇	0.40	0.50	0.40	0.50	1.00	0.00	0.85	0.10	0.50	0.40	0.10	0.90	0.70	0.20
E ₈	0.50	0.40	0.70	0.20	0.10	0.75	0.70	0.20	0.40	0.50	0.85	0.10	0.60	0.30
E ₉	0.70	0.20	0.10	0.90	1.00	0.00	1.00	0.00	1.00	0.00	0.60	0.30	0.80	0.15
E ₁₀	0.25	0.60	0.40	0.50	0.85	0.10	0.50	0.40	0.70	0.20	1.00	0.00	0.60	0.30
E ₁₁	0.25	0.60	0.70	0.20	0.10	0.90	0.50	0.40	0.60	0.30	0.40	0.50	1.00	0.00
E ₁₂	0.10	0.75	0.80	0.15	0.85	0.10	0.25	0.60	0.80	0.15	1.00	0.00	0.50	0.40
E ₁₃	0.85	0.10	0.10	0.75	0.50	0.40	1.00	0.00	0.40	0.50	1.00	0.00	1.00	0.00
E ₁₄	0.60	0.30	0.10	0.75	1.00	0.00	0.60	0.30	1.00	0.00	0.70	0.20	0.80	0.15
E ₁₅	0.50	0.40	0.85	0.10	0.85	0.10	0.50	0.40	1.00	0.00	0.60	0.30	0.60	0.30
E ₁₆	0.10	0.90	1.00	0.00	0.80	0.15	0.10	0.90	1.00	0.00	0.10	0.75	0.10	0.90

Subsequently, the magnitude of the max and min components is calculated using *Eqs. (16)-(19)*, and is reported in *Table 10*. For example, the max component for experimental run 1 is computed using IF operation laws in *Eqs. (3)* and *(6)* as $\left(((0.075)^2)^{\frac{1}{2}}, (1 - (1 - 0.919)^2)^{\frac{1}{2}} \right) = \langle 0.075, 0.919 \rangle$. Since MRR is the only beneficial

criterion, the max component \tilde{U}_{ik} consists of only MRR. On the other hand, the min component consists of all the non-beneficial criteria, i.e., TWR, SR, OC, TC, CIR, and CYL. Therefore, \tilde{U}_{ih} for experiment 1 is calculated as $\left((1 - (1 - 0.731^2)(1 - 0.731^2)(1 - 0.603^2)(1 - 0.060^2)(1 - 0.454^2)(1 - 0.567^2))^{1/2}, (1 - (1 - 0.213^2 \times 0.213^2 \times 0.343^2 \times 0.934^2 \times 0.471^2 \times 0.378^2))^{1/2} \right) = \langle 0.962, 0.018 \rangle$. Using the magnitudes of the components, the median of the optimal alternative is estimated based on *Eq. (20)*, and the median of the individual alternatives is determined by applying *Eq. (21)*. For experiment 1, $\tilde{U}_{1k}^2 + \tilde{U}_{1h}^2 = (1 - (1 - 0.075^2)(1 - 0.962^2), (1 - (1 - 0.919)^2)(1 - (1 - 0.018)^2)) = \langle 0.926, 0.036 \rangle$. Therefore, the median for experiment 1 can be calculated as $\left(1 - (1 - 0.926^{1/2}), (1 - (1 - 0.036)^{1/2}) \right) = \langle 0.806, 0.135 \rangle$ using *Eqs. (3), (5) and (6)*.

Table 9. Weighted IF decision matrix and optimal alternative for Example 1.

Exp. No.	SR		MRR		TWR		OC		TC		CIR		CYL	
	μ	ν												
E ₁	0.731	0.213	0.075	0.919	0.731	0.213	0.603	0.343	0.060	0.934	0.454	0.471	0.567	0.378
E ₂	0.512	0.370	0.188	0.677	0.439	0.449	0.483	0.442	0.151	0.737	0.482	0.440	0.454	0.471
E ₃	0.292	0.606	0.377	0.516	0.439	0.449	0.422	0.475	0.302	0.606	0.567	0.378	0.283	0.627
E ₄	0.183	0.685	0.188	0.677	0.731	0.213	0.483	0.442	0.302	0.606	0.567	0.378	0.397	0.502
E ₅	0.621	0.291	0.075	0.798	0.621	0.291	0.483	0.442	0.513	0.409	0.283	0.627	0.057	0.938
E ₆	0.621	0.291	0.075	0.798	0.731	0.213	0.603	0.343	0.060	0.934	0.567	0.378	0.454	0.471
E ₇	0.292	0.606	0.301	0.597	0.731	0.213	0.513	0.409	0.302	0.606	0.057	0.938	0.397	0.502
E ₈	0.365	0.528	0.527	0.354	0.073	0.803	0.422	0.475	0.241	0.672	0.482	0.440	0.340	0.565
E ₉	0.512	0.370	0.075	0.919	0.731	0.213	0.603	0.343	0.603	0.343	0.340	0.565	0.454	0.471
E ₁₀	0.183	0.685	0.301	0.597	0.621	0.291	0.302	0.606	0.422	0.475	0.567	0.378	0.340	0.565
E ₁₁	0.183	0.685	0.527	0.354	0.073	0.921	0.302	0.606	0.362	0.540	0.227	0.689	0.567	0.378
E ₁₂	0.073	0.803	0.603	0.314	0.621	0.291	0.151	0.737	0.483	0.442	0.567	0.378	0.283	0.627
E ₁₃	0.621	0.291	0.075	0.798	0.365	0.528	0.603	0.343	0.241	0.672	0.567	0.378	0.567	0.378
E ₁₄	0.439	0.449	0.075	0.798	0.731	0.213	0.362	0.540	0.603	0.343	0.397	0.502	0.454	0.471
E ₁₅	0.365	0.528	0.640	0.274	0.621	0.291	0.302	0.606	0.603	0.343	0.340	0.565	0.340	0.565
E ₁₆	0.073	0.921	0.753	0.193	0.585	0.331	0.060	0.934	0.603	0.343	0.057	0.844	0.057	0.938
\bar{Q}	0.731	0.213	0.753	0.193	0.731	0.213	0.603	0.343	0.603	0.343	0.567	0.378	0.567	0.378

Similarly, it can be shown that the value of the median for the optimal experimental run is $\langle 0.906, 0.055 \rangle$. Finally, the median similarity between each experiment and the optimal experiment is determined using *Eq. (22)*, and the ranks of the experiments are obtained in the decreasing order of the median similarity measure. For example, median similarity for experiment 1 is obtained by calculating the scores for the median of experiment 1 and the optimal alternative as $\frac{0.806 - 0.135 + 1}{2} = 0.836$ and $\frac{0.906 - 0.055 + 1}{2} = 0.925$, respectively. Next, the median similarity is calculated as the ratio $\frac{0.836}{0.925} = 0.903$.

Table 10 shows the median of each experimental run, its corresponding median similarity, and the final ranks of the different EDM experiments conducted. The results reveal that experimental run 1 with $I = 3A$, $T_{ON} = 10 \mu s$, $T_{OFF} = 5 \mu s$ and copper tool material is the desired intermix of EDM parameters resulting in SR = 3.20 μm , MRR = 7.5512 mm^3/min , TWR = 0.0042 g/min , OC = 0.0140 mm , TC = 0.061 mm , CIR = 0.0184 mm , and CYL = 0.0421 mm . Experimental run 9 has the second most-ideal settings for the EDM parameters, and experiment 11 has the worst response values.

Table 10. The magnitude of the decomposed components for Example 1.

Exp. No.	\tilde{U}_{ik}		\tilde{U}_{ih}		\tilde{M}_i		MS_i	Rank
	μ	ν	μ	ν	μ	ν		
E ₁	0.075	0.919	0.962	0.018	0.806	0.135	0.903	1
E ₂	0.188	0.677	0.853	0.069	0.624	0.248	0.743	13
E ₃	0.377	0.516	0.810	0.100	0.599	0.275	0.716	14
E ₄	0.188	0.677	0.906	0.047	0.699	0.205	0.807	7
E ₅	0.075	0.798	0.897	0.049	0.680	0.216	0.791	9
E ₆	0.075	0.798	0.950	0.024	0.777	0.152	0.878	3
E ₇	0.301	0.597	0.872	0.068	0.660	0.239	0.768	11
E ₈	0.527	0.354	0.740	0.145	0.576	0.286	0.697	15
E ₉	0.075	0.919	0.950	0.022	0.777	0.147	0.880	2
E ₁₀	0.301	0.597	0.857	0.071	0.640	0.242	0.755	12
E ₁₁	0.527	0.354	0.715	0.182	0.558	0.319	0.669	16
E ₁₂	0.603	0.314	0.845	0.084	0.691	0.209	0.800	8
E ₁₃	0.075	0.798	0.924	0.038	0.725	0.190	0.829	6
E ₁₄	0.075	0.798	0.928	0.033	0.732	0.177	0.840	5
E ₁₅	0.640	0.274	0.871	0.063	0.729	0.171	0.841	4
E ₁₆	0.753	0.193	0.767	0.165	0.694	0.233	0.790	10
\tilde{Q}	0.753	0.193	0.980	0.009	0.906	0.055		

Next, the same EDM process is considered, and the ideal choice of process parameters is found using the IF-RATMI method. Based on the magnitude of the max and min components of the alternatives, elements of the diagonal product matrix and its corresponding trace are respectively computed by applying *Eqs. (25)* and *(26)*. To demonstrate the mathematical working, the trace for experiment 1 is calculated as the score of the IFN $\langle 1 - (1 - 0.075 \times 0.753)(1 - 0.962 \times 0.980), (0.919 + 0.193 - (0.919 \times 0.193)) \times (0.018 + 0.009 - (0.018 \times 0.009)) \rangle = \langle 0.946, 0.025 \rangle$, i.e., $\frac{0.946 - 0.025 + 1}{2} = 0.960$.

Utilizing the values in *Table 10* and *Eqs. (27)-(29)*, the majority index for all the experimental runs is evaluated. Subsequently, the IF-RATMI method is concluded by ranking the experiments while sorting the majority index in a decreasing manner, as evidenced by *Table 11*. For experiment 1, the majority index is $0.5 \times \frac{0.903 - 0.669}{0.903 - 0.669} + 0.5 \times \frac{0.960 - 0.864}{0.960 - 0.864} = 1$. Here, the denominator uses the maximum and minimum values for trace and median similarity obtained from *Tables 10* and *11*. The rankings obtained by the experimental runs using the IF-RATMI method demonstrate a strong similarity to those of the IF-RAMS method. The best combination of the EDM parameters is present in experimental run 1. On the other hand, experiment 9 has the second most-optimal values of the process parameters, while experiment number 11 proves to incorporate the worst parametric setting for the considered EDM process.

Table 11. Final ranks of experimental runs using the IF-RATMI approach for Example 1.

Exp. No.	T_i	tr_i	E_i	Rank
E ₁	0.946	0.025	0.960	1.000
E ₂	0.859	0.057	0.901	0.348
E ₃	0.852	0.066	0.893	0.248
E ₄	0.903	0.041	0.931	0.642
E ₅	0.885	0.048	0.919	0.543
E ₆	0.934	0.027	0.954	0.911
E ₇	0.887	0.052	0.918	0.488

Table 11. Continued.

Exp. No.	T _i	tr _i	E _i	Rank	Exp. No.
E ₈	0.834	0.073	0.881	0.143	15
E ₉	0.934	0.029	0.953	0.912	2
E ₁₀	0.876	0.053	0.911	0.428	12
E ₁₁	0.819	0.091	0.864	0.000	16
E ₁₂	0.906	0.041	0.932	0.634	8
E ₁₃	0.910	0.039	0.936	0.714	6
E ₁₄	0.914	0.035	0.940	0.758	5
E ₁₅	0.924	0.030	0.947	0.800	4
E ₁₆	0.892	0.060	0.916	0.527	10

To verify the validity of the derived results, comparative analyses of IF-RAMS and IF-RATMI are conducted with IF extensions of other well-known MCDM techniques, like Weighted Aggregated Sum Product ASsessment (WASPAS), TOPSIS, PROMETHEE, and Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) [45], [46]. The rankings of the experiments obtained through IF-RAMS and IF-RATMI are individually compared with those obtained using the four above-mentioned IF-MCDM approaches, and the corresponding values of Spearman's rank correlation (ρ) and Kendall's Tau (τ) coefficients are computed. It is interestingly noticed that both IF-RAMS and IF-RATMI have $\rho = 0.703, 0.815, 0.865$, and 0.968 , and $\tau = 0.567, 0.633, 0.733$, and 0.917 with IF-WASPAS, IF-TOPSIS, IF-PROMETHEE and IF-MOORA, respectively.

Furthermore, IF-RAMS and IF-RATMI have a perfect positive correlation between them with respect to both ρ and τ values. Finally, sensitivity analysis studies are conducted to prove the consistency and robustness of the rankings for IF-RAMS and IF-RATMI subject to variations in response weights assigned by different stakeholders (decision-makers). To that end, 31 unique scenarios are generated by varying the criteria weights assigned to each criterion by the stakeholders. A comprehensive graphical illustration of the outcomes of the sensitivity analysis study is shown in *Fig. 3*. From *Fig. 3* it can be noted that experimental run 1 achieves rank 1 in 21 scenarios (67.7%) for IF-RAMS and in 20 scenarios (64.5%) for IF-RATMI.

Furthermore, in both methods, experiment 9 is selected to have the best combination of parameters in 8 scenarios (25.8%), while experiment 11 is found to have the worst parametric combination in 19 scenarios (61.3%). Finally, both methods rank experiment 6 as having the optimal parametric setting in 2 scenarios (6.5%) and the second-best parametric setting in 10 scenarios (32.3%). Therefore, from the observations of the sensitivity analysis study, it can be concluded that both IF-RAMS and IF-RATMI are sufficiently consistent and robust in searching out the best and worst parametric combinations for the said EDM process, regardless of the subjectivity in the stakeholders' opinions.

From the results derived using both IF-RAMS and IF-RATMI methods, it can be stated that lower values of the considered EDM factors, particularly I, T_{ON}, and T_{OFF} are preferable to simultaneously increase the MRR, and lower the TWR, SR, OC, TC, CIR, and CYL. Although increasing peak current would increase MRR as more material would be vaporized by stronger electrical discharges, but increase in the size of the craters formed would drastically deteriorate the surface quality of the machined components. A similar trend is also observed for T_{ON} where its higher value would result in higher MRR, but would significantly reduce surface quality and dimensional accuracy.

Furthermore, increasing I and T_{ON} would also increase tool wear due to an increase in the intensity of electrical discharges, causing higher removal of tool electrode material [47]. Hence, the dimensional accuracy of the finished products is adversely affected by higher values of I and T_{ON}. Due to higher energy discharge during sparking, more material would get removed from the workpiece, resulting in higher overcut values [48]. Tapers in the machined holes would also increase with an increase in pulse duration, resulting in poor dimensional accuracy [49]. Therefore, to achieve a balance between productivity, surface quality, and

dimensional accuracy of the final products, it is recommended that the input parameters during the EDM operation of AZ31 magnesium alloy be set to low values.

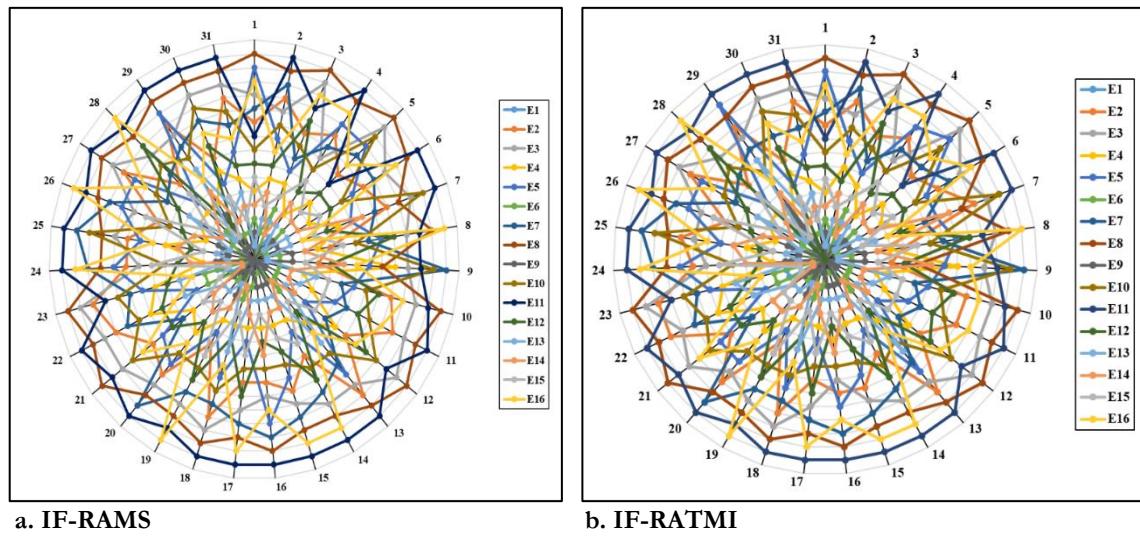


Fig. 3. Results of sensitivity analysis study for IF-RAMS and IF-RATMI methods.

Example 2. To strengthen the applicability and potentiality of IF-RAMS and IF-RATMI approaches in optimizing EDM processes, the experimental data from Phan et al. [50] is treated here as the second illustrative example. Using Taguchi's L_{16} OA, Phan et al. [50] performed 16 EDM experiments on Ti-6Al-4V work material to examine the influences of I , V_g , and T_{ON} on MRR, TWR, and SR. The Ti-6Al-4V workpieces were machined using a die-sinking ZNC-type EDM setup (Electronics India Private Limited) with thin film AlCrNi-coated aluminium electrodes.

During the experiments, four different levels are considered for each of the EDM parameters, i.e., $I = 10, 20, 30$, or 40 A, $V_g = 40, 45, 50$, or 55 V and $T_{ON} = 100, 500, 1000$, or 1500 μ s. The variation in the mass of the two electrodes (workpiece and tool) is noted (employing a high-precision Contech weight balance) to estimate the MRR and TWR values, respectively, for each experimental run. Similarly, the SR of the machined components was recorded using the taylor hobson surtronic S-100 series surface roughness testing machine (cut-off length 0.8 mm). Using the measured response values, Phan et al. [50] adopted the Taguchi-DEAR method to seek the ideal combination of parameters for the EDM process. The results indicated that $I = 40$ A, $V_g = 55$ V, and $T_{ON} = 1000$ μ s would provide the optimal intermix of the considered EDM parameters. The responses measured after the different experimental trials are displayed in *Table 12*.

Table 12. Experimental data for Example 2 [50].

Exp. No.	I (A)	V_g (V)	T_{ON} (μ s)	MRR (g/min)	TWR (g/min)	SR(μ m)
E ₁	10	40	100	0.004	0.0001	6.111
E ₂	10	45	500	0.0093	0.0002	6.162
E ₃	10	50	1000	0.0086	0.0002	6.251
E ₄	10	55	1500	0.0065	0.0003	6.294
E ₅	20	45	100	0.0053	0.0014	6.874
E ₆	20	40	500	0.0107	0.0022	6.976
E ₇	20	55	1000	0.0089	0.0011	7.648
E ₈	20	50	1500	0.0148	0.0022	7.669
E ₉	30	50	100	0.0077	0.0031	8.045
E ₁₀	30	55	500	0.0106	0.0041	8.338
E ₁₁	30	40	1000	0.0098	0.0037	8.768
E ₁₂	30	45	1500	0.0099	0.0046	8.946
E ₁₃	40	55	100	0.0133	0.0074	9.013
E ₁₄	40	50	500	0.0125	0.0081	9.112
E ₁₅	40	45	1000	0.0139	0.0089	9.313
E ₁₆	40	40	100	0.0121	0.0095	9.623

To optimize the said EDM process using IF-RAMS and IF-RATMI tools, the evaluation of the importance of the responses relative to each other is a crucial step. The responses are first evaluated in linguistic terms by the three decision-makers. The manufacturer prioritizes productivity by assigning the highest importance to MRR.

The process engineer wants to have minimal tool wear and assigns TWR the highest relative importance, while the end-user provides maximum importance to product quality by evaluating SR as the most important response. Based on the linguistic ratings provided in *Table 13*, along with the conversion scale shown in *Table 3*, the aggregate IF criteria weights are computed and presented in *Table 14*.

Table 13. Linguistic evaluation of responses by the stakeholders for Example 2.

Response	Manufacturer	Process Engineer	End-User
MRR	VI	MI	I
TWR	MI	VI	MI
SR	I	I	VI

Table 14. Aggregated IF response weights for Example 2.

Response	μ	ν	π
MRR	0.753	0.193	0.054
TWR	0.689	0.253	0.058
SR	0.804	0.147	0.048

Considering the experimental data from *Table 12* and the scale described in *Table 15*, the experiments' performance in relation to each of the responses is transformed into linguistic terms. Next, the linguistic performance ratings are converted into IFNs by employing the conversion system described in *Table 7* to develop the initial IF decision matrix.

Table 15. Scale for linguistic evaluation of experimental runs for Example 2.

Linguistic Variable	MRR	TWR	SR
EG	$MRR > 0.01372$	$TWR \leq 0.00104$	$SR \leq 6.4622$
VVG	$0.01264 < MRR \leq 0.01372$	$0.00104 < TWR \leq 0.00198$	$6.4622 < SR \leq 6.8134$
VG	$0.01153 < MRR \leq 0.01264$	$0.00198 < TWR \leq 0.00292$	$6.8134 < SR \leq 7.1646$
G	$0.01048 < MRR \leq 0.01153$	$0.00292 < TWR \leq 0.00386$	$7.1646 < SR \leq 7.5158$
MG	$0.0094 < MRR \leq 0.01048$	$0.00386 < TWR \leq 0.0048$	$7.5158 < SR \leq 7.867$
M	$0.00832 < MRR \leq 0.0094$	$0.0048 < TWR \leq 0.00574$	$7.867 < SR \leq 8.2182$
MB	$0.00724 < MRR \leq 0.00832$	$0.00574 < TWR \leq 0.00668$	$8.2182 < SR \leq 8.5674$
B	$0.00616 < MRR \leq 0.00724$	$0.00668 < TWR \leq 0.00762$	$8.5694 < SR \leq 8.9206$
VB	$0.00508 < MRR \leq 0.00616$	$0.00762 < TWR \leq 0.00856$	$8.9206 < SR \leq 9.2718$
VVB	$MRR \leq 0.00508$	$TWR > 0.00856$	$SR > 9.2718$

As presented in the first example, IF-RAMS and IF-RATMI methods are also applied here to derive the final ranks of the experiments that were conducted. Based on the rankings computed in *Table 16*, it is interestingly revealed that experimental runs 2 and 3 are tied at rank 1 in both approaches. Experimental number 2, having a combination of different EDM parameters as $I = 10A$, $V_g = 45V$, and $T_{ON} = 500 \mu s$ provides the corresponding response values as $MRR = 0.0093 \text{ g/min}$, $TWR = 0.002 \text{ g/min}$, and $SR = 6.162 \mu m$.

On the other hand, trial number 3 results in $MRR = 0.0086 \text{ g/min}$, $TWR = 0.0002 \text{ g/min}$, and $SR = 6.251 \mu m$ at a parametric mix of $I = 10A$, $V_g = 50 V$, and $T_{ON} = 1000 \mu s$. By comparing the responses of experiment runs 2 and 3, it can be noticed that run 2 has a slightly higher value for MRR and a lower value for SR. Since only ten linguistic variables are considered here to evaluate the performance ratings of the conducted experiments, both IF-RAMS and IF-RATMI assign the same linguistic terms to experimental runs 2 and 3, resulting in tied ranks. Furthermore, experimental run 16 has the least favourable setting of EDM parameters in both methods.

Table 16. Final ranking of the experimental runs using IF-RAMS and IF-RATMI for Example 2.

Exp. No.	MS _i	IF-RAMS	tr _i	E _i	IF-RATMI
E ₁	0.872	4	0.856	0.901	4
E ₂	0.905	1	0.897	1.000	1
E ₃	0.905	1	0.897	1.000	1
E ₄	0.883	3	0.876	0.942	3
E ₅	0.709	7	0.781	0.593	7
E ₆	0.791	6	0.859	0.805	6
E ₇	0.679	8	0.795	0.576	8
E ₈	0.839	5	0.871	0.882	5
E ₉	0.563	13	0.714	0.317	12
E ₁₀	0.604	10	0.740	0.404	9
E ₁₁	0.567	11	0.719	0.329	10
E ₁₂	0.505	15	0.664	0.173	14
E ₁₃	0.564	12	0.649	0.223	13
E ₁₄	0.509	14	0.589	0.068	15
E ₁₅	0.648	9	0.648	0.325	11
E ₁₆	0.495	16	0.554	0.000	16

From the ranks obtained in *Table 16*, it is revealed that to achieve the desired balance between surface quality and productivity, it is preferable to assign low-to-medium values to all the EDM parameters. As demonstrated in the previous example, higher values of I, V_g, and T_{ON} would result in higher energy discharge during the sparking process. Although the MRR of the process would increase, higher energy electrical discharge would lead to higher tool wear and poorer surface finish. Therefore, to simultaneously optimize MRR, TWR, and SR, it is always necessary to ensure that the values of I, V_g and T_{ON} should not be too high.

Furthermore, to prove the validity of the results and recommendations above, similar comparative analyses are also conducted here, as mentioned in the previous example. For this example, IF-RAMS achieves $\varrho = 0.918$, 0.841, 0.910, and 0.9363, and $\tau = 0.782$, 0.647, 0.765, and 0.866 with IF-WASPAS, IF-TOPSIS, IF-PROMETHEE, and IF-MOORA, respectively.

The corresponding values for IF-RATMI are evaluated as $\varrho = 0.952$, 0.884, 0.949, and 0.9386, and $\tau = 0.849$, 0.714, 0.832, and 0.933, respectively. Therefore, owing to the high correlation between the rankings obtained through IF-RAMS and IF-RATMI and those derived using the four well-known MCDM approaches (IF-WASPAS, IF-TOPSIS, IF-PROMETHEE, and IF-MOORA), it is concluded that both the proposed approaches can provide reliable and accurate ranking results. Finally, as described in the previous example, similar sensitivity analysis studies are also conducted for this example.

Through further analysis of the sensitivity analysis studies, it can be unveiled that experiments 2 and 3 are tied at rank 1 in 23 scenarios (74.2%) for IF-RAMS and 21 scenarios (67.7%) for IF-RATMI, whereas they achieve the second-best parametric combination in 8 scenarios (25.8%) and 10 scenarios (32.3%) for IF-RAMS and IF-RATMI, respectively. Finally, experiment 16 has the worst combination of EDM parameters for 25 scenarios (80.6%) with IF-RAMS and all 31 scenarios (100%) for IF-RATMI. Therefore, the consistency and robustness of both IF-RAMS and IF-RATMI are further verified based on the sensitivity analysis results. Increased consistency of results in *Example 2* as compared to *Example 1* is associated with the presence of an increased number of decision criteria (sources of variability) in *Example 1*.

5 | Conclusion

In this paper, two newly developed MCDM tools, specifically the RAMS and RATMI methods, are applied for the first time in an IF environment to obtain the optimal process parameter settings of two EDM processes. In real-world manufacturing environments, there exists a large amount of subjectivity in decision-making owing to the disparity in the personal preferences of different contributors (manufacturer, process engineer, end-user, etc.), leading to biased evaluations by decision-making experts. IFSs emerge as a competent means of tackling the underlying ambiguity and uncertainty associated with decision-making in such scenarios.

While performing EDM of biodegradable AZ31 magnesium alloy, both IF-RAMS and IF-RATMI methods identify $I = 3A$, $T_{ON} = 10 \mu s$, $T_{OFF} = 5 \mu s$ and copper tool material as the optimal combination of input parameters that results in $SR = 3.20 \mu m$, $MRR = 7.5512 mm^3/min$, $TWR = 0.0042 g/min$, $OC = 0.0140 mm$, $TC = 0.061 mm$, $CIR = 0.0184 mm$, and $CYL = 0.0421 mm$. It can be interestingly noted that these results are consistent with those obtained by past researchers employing GRA and TOPSIS.

During the EDM of Ti-6Al-4V alloy workpieces, both IF-RAMS and IF-RATMI result in a tie between two parametric combinations. The EDM experiment with $I = 10A$, $V_g = 45V$, and $T_{ON} = 500 \mu s$ lead to slightly better values for MRR and SR compared to the experiment having a combination of EDM parameters as $I = 10A$, $V_g = 50 V$ and $T_{ON} = 1000 \mu s$. However, IF-RAMS and IF-RATMI rank them equally as the optimal solutions due to the limitation of the linguistic terms in accounting for very small changes in the response values. Therefore, it can be concluded that both IF-RAMS and IF-RATMI can effectively identify the ideal combination of input parameters for any of the EDM processes that would be consistent with other popular MCDM techniques.

Furthermore, comparative analyses with IF extensions of some well-known MCDM methods reveal strong positive Spearman's rank correlation as well as Kendall's Tau values between the rankings obtained through IF-RAMS and IF-RATMI, and those obtained employing IF-WASPAS, IF-TOPSIS, IF-PROMETHEE, and IF-MOORA. The corresponding sensitivity analysis studies also reiterate the consistent and robust decision-making ability of the adopted approaches. Hence, it can be concluded that IF-RAMS and IF-RATMI are valid MCDM tools that may be effectively utilized in scenarios with conflicting criteria and uncertainty in information. Further research may be focused on extending RAMS and RATMI approaches in different fuzzy environments, namely neutrosophic fuzzy, hesitant fuzzy, spherical fuzzy, picture fuzzy, etc., as well as integrating them with grey system theory, soft sets, and rough sets to solve for the ideal level of input parameters for various machining processes.

Author Contribution

Conceptualization, S.C.; Methodology, S.C.; Formal Analysis, S.C.; Investigation, S.C.; Validation, S.C., and Sh.C.; Writing-creating the initial design, S.C., and Sh.C.; Writing-reviewing and editing, Sh.C.

Funding

No specific funding has been required.

Data Availability

All data analysed during this study are included in the text.

Conflict of Interest

The authors declare that they have no competing interests.

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